FAQ and erratum to “Radon partitions in convexity spaces”

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What is the publication status of the paper?

I received a thorough referee report, which justly complained about numerous minor issues. The referee also recommended to keep only the counterexample to Eckhoff’s conjecture, and to remove the upper bound. I should have simply said “no” to the latter demand, and revised the paper carefully. Instead, because of personal reasons, I let the whole paper lay dormant. That was a mistake.

The editorial decision was “accepted subject to revisions”, which is why the paper is listed as “accepted” on my webpage.

Are you planning to revise the paper?

No. I am no longer actively working in the area of abstract convexity, and I have no plans of revising the paper.

Is the paper correct?

Despite being poorly written, the counterexample to Eckhoff’s conjecture is correct, as far as I know.

As pointed by Kaarel Hänni, the proof Lemma 13 is incorrect without a condition that the singleton sets of $(X, \mathcal{F})$ are convex. Similarly, proof of Proposition 4 also implicitly assumes that singletons are convex. Without this condition, the proof is incorrect (though the result itself is true).

What progress has there been on the problems in the paper?

Holmsen and Lee [1] showed existence of bounded-sized weak ε-nets in spaces with bounded Radon number, thereby answering Question 3 in positive.

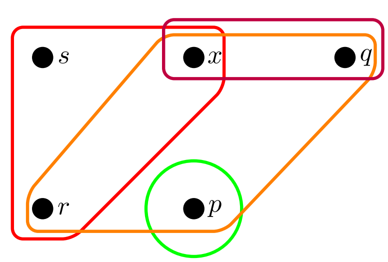
Pálvölgyi [2] improved Theorem 1, by showing that, for fixed r_2, the Radon numbers grow linearly in k.

I managed to simplify/clean this paper.

Great! Please make your simplification public. This is a badly written paper, and it would benefit from a better exposition.

What is wrong with Lemma 13?

Consider the following example (courtesy of Kaarel Hänni):



Then $\{q, r\}$ is in any maximal family containing $\{p\}$, and $\{r, s\}$ is in any maximal family containing $\{q\}$, but there is no maximal family containing $\{p\}$ that also contains $\{r, s\}$.

So what is the correct proof of Proposition 4?

Here it is (again courtesy of of Kaarel Hänni).

A convex set is *minimal* if it contains no proper non-empty convex subset. A point is minimal, if its convex hull is minimal. Observe that for every three minimal points, one is in convex hull of the other two (since $r_2 = 3$).

For each $x \in X$, pick a minimal point in $\operatorname{conv}(x)$, and denote it $d(x)$. If there is more than one such minimal point, choose arbitrarily. Say that x is *covered* by $\{y, z\}$ if $d(x) \in \operatorname{conv}\big(d(y), d(z)\big)$. By the preceding observation, among any three points, one is covered by the other two. So, let x be a point covered by at least $\frac{1}{3}\binom{n}{2}$ pairs. Since $\operatorname{conv}\big(d(y), d(z)\big) \subseteq \operatorname{conv}\big(y, z\big)$ always holds, it follows that $p = d(x)$ is the desired point.

References

[1] Dong-Gyu Lee Andreas F. Holmsen. Radon numbers and the fractional helly theorem. arXiv:1903.01068.

[2] Dömötör Pálvölgyi. Radon numbers grow linearly. arXiv:1912.02239.