Discrete Math: homework #7*
Due 1 December 2021, at 9:00am

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail. I want both the LaTeX file and the resulting PDF. The files must be of the form `andrewid_discr_hwnum.tex` and `andrewid_discr_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

You can earn 0.5 point of extra credit by submitting exactly 1 problem before 9am on 24 November 2021. Resubmissions void the extra credit.

1. [2] Let $k \notin \{1,3\} \pmod 6$. Suppose $\mathcal{F} \subset \binom{[n]}{k}$ is $\{0,1,3\}$-intersecting. Show that $|\mathcal{F}| \le cn^2$, where the constant c is independent of k. [To solve this problem you do *not* need sunflowers.]

2. [1+1]

   (a) Let $\mathcal{F}$ be the family consisting of all the halfspaces in $\mathbb{R}^d$. Prove that the VC-dimension of $\mathcal{F}$ is $d+1$. [Hint: look at your notes for September.]

   (b) Let $\mathcal{F}$ be the family of consisting of all axis-parallel boxes in $\mathbb{R}^d$, i.e., the sets of the form $I_1 \times I_2 \times \cdots \times I_d$, where each I_j is an interval. Show that the VC-dimension of $\mathcal{F}$ is $2d$.

3. [1+1] Let P be a set of n points in $\mathbb{R}^2$.

   (a) Prove that P spans at most $O(n^{7/3})$ triangles of unit area.

   (b) Prove that P spans at most $O(n^{7/3})$ triangles with a given fixed angle α.

*This homework is from `http://www.borisbukh.org/DiscreteMath21/hw7.pdf`.