Algebraic Structures: homework #9
Due 4 November 2024, at 9am via Gradescope

To receive full credit, all work must be shown. A passage means what careful but unimaginative reader thinks it does. Add details if in doubt. The problems should be written neatly and in order they were assigned.

A typical homework assignment is graded out of 20 points: 4 points for correctness of each problem. Bonus points result in additional credit.

0. (Ungraded)
   - Prepare for voting on Tuesday, November 5th. You will live longer, it matters more to you.
   - Finish reading Section 3.7; this is what we covered by the 9th week. Did you find any mistakes or typos? If you did not, you might not have read carefully enough.
   - Continue reading Chapter 3.
1. Problem 2 on page 140. [Consider solving the preceding problem first.]
2. Problem 4 on page 142.
3. Problem 7 on page 143.
4. Let R be a commutative ring, and I is an ideal in R. Show that the set
   $$\{r \in R : \exists n \in \mathbb{Z}\, n > 0,\ r^n \in I\}$$
   is an ideal in R.
5. Give an example of rings R_1, R_2 and R_3 such that
   - R_1 is a subring of R_2,
   - R_2 is a subring of R_3,
   - both R_1 and R_3 are Euclidean rings

- R_2 is not Euclidean

Explain why your example satisfies the conditions. [One of the possible approaches: show that the set of polynomials with vanishing linear coefficient is a ring, and that this ring contains a non-principal ideal.]