

Algebraic Structures: homework #7

Due 21 October 2024, at 9am via Gradescope

To receive full credit, all work must be shown. A passage means what careful but unimaginative reader thinks it does. Add details if in doubt. The problems should be written neatly and in order they were assigned.

A typical homework assignment is graded out of 20 points: 4 points for correctness of each problem. Bonus points result in additional credit.

0. (Ungraded)

- Finish reading Chapter 2; this is what we covered by the 7th week. Did you find any mistakes or typos? If you did not, you might not have read carefully enough.
- Start reading Chapter 3.

1. Problem 5 on page 102. [Reading textbook is helpful for this.]

2. Problem 8 on page 108.

3. Problem 6 on page 108. [Also look at Problem 7 below, but it is not graded.]

4. Problem 4 on page 115.

5. Problem 6 on page 115, under the extra assumption that $k \leq 2$.

6. (Bonus; 2pt) Let $G = \underbrace{\mathbb{Z}/2\mathbb{Z} \times \cdots \times \mathbb{Z}/2\mathbb{Z}}_{n \text{ factors}}$ (the usual notation is $(\mathbb{Z}/2\mathbb{Z})^n$

for this group). Consider also $\mathbb{R} \setminus \{0\}$ with the usual multiplication. Let \hat{G} denote the set of homomorphisms from G to $\mathbb{R} \setminus \{0\}$. Show that \hat{G} is a group under the usual multiplication of real-valued functions, and that $\hat{G} \approx G$. [If you are curious about more general abelian groups, look at exercises 7–12 on page 115.]