Algebraic Structures: homework #5
Due 30 September 2024, at 9am via Gradescope

To receive full credit, all work must be shown. A passage means what careful but unimaginative reader thinks it does. Add details if in doubt. The problems should be written neatly and in order they were assigned.

A typical homework assignment is graded out of 20 points: 4 points for correctness of each problem. Bonus points result in additional credit.

0. (Ungraded)
   - Finish reading Chapter 2 through section 2.11; this is what we covered by the 5th week. Did you find any mistakes or typos? If you did not, you might not have read carefully enough.
   - Continue reading Chapter 2.
1. Problem 11 on page 81. [The main difficulty in this problem is not to come up with a solution, but to avoid handwaving when explaining it.]
2. (a) Problem 3 on page 80.
   (b) Problem 7 on page 80.
3. Let p be a prime. How many subgroups of S_p have order p?
4. (a) Show that every element of S_n is a product of at most $n-1$ transpositions.
   (b) Show that every element of S_n is a product of at most $n/2$ transpositions and 3-cycles in total.
5. Compute the order of $\mathcal{A}(G)$, for the following groups. Note that some of these might be infinite; in these cases, specify if $o(\mathcal{A}(G))$ is countable or not.
   (a) $G = \mathbb{Q}$.
   (b) $G = \mathbb{R}^2$.
6. (Bonus; 2pt) Prove that every permutation in S_n can be written as a product of two cycles (of any lengths).