

Algebraic Structures: homework #4

Due 23 September 2024, at 9am via Gradescope

To receive full credit, all work must be shown. A passage means what careful but unimaginative reader thinks it does. Add details if in doubt. The problems should be written neatly and in order they were assigned.

A typical homework assignment is graded out of 20 points: 4 points for correctness of each problem. Bonus points result in additional credit.

0. (Ungraded)

- Finish reading Chapter 2 through section 2.8; this is what we covered by the 4th week. Did you find any mistakes or typos? If you did not, you might not have read carefully enough.
- Continue reading Chapter 2.

1. Suppose K and L are normal subgroups of G . Show that $K \cap L$ is a normal subgroup of G .
2. Problem 1 on page 70.
3. Problem 4 on pages 74-75.
4. Let H, K be subgroups of a group G . Show that $i_G(H \cap K) \leq i_G(H)i_G(K)$. [Be careful: G need not be finite.]
5. (a) Prove that the groups \mathbb{Z} and \mathbb{Q} are not isomorphic.
(b) Prove that the groups $\text{GL}_2(\mathbb{R})$ and \mathbb{R}^3 are not isomorphic.