Algebraic Structures: homework #3
Due 16 September 2024, at 9am via Gradescope

To receive full credit, all work must be shown. A passage means what careful but unimaginative reader thinks it does. Add details if in doubt. The problems should be written neatly and in order they were assigned.

A typical homework assignment is graded out of 20 points: 4 points for correctness of each problem. Bonus points result in additional credit.

0. (Ungraded)
   - Finish reading Chapter 2 through section 2.6; this is what we covered by the 3rd week. Did you find any mistakes or typos? If you did not, you might not have read carefully enough.
   - Continue reading Chapter 2.
1. Let G be a finite abelian group such that, for every positive integer n, the equation $x^n = e$ has at most n solutions.
   (a) Prove that for every two elements $a, b \in G$, there is a $c \in G$ such that $a, b \in (c)$.
   (b) Use part (a) to solve problem 38 on page 49.
2. Problem 8 on page 53.
3. Problem 13 on page 53.
4. Problem 12 on page 53.
5. Problem 18 on page 54.
6. (Bonus; 2pts + 2pts) Let G be the group of rational numbers under the usual addition.
   (a) Show that the only subgroup of G of finite index is G itself.
   (b) Show that G contains uncountably many distinct subgroups.