

# Algebraic Structures: homework #2

## Due 9 September 2024, at 9am via Gradescope

To receive full credit, all work must be shown. A passage means what careful but unimaginative reader thinks it does. Add details if in doubt. The problems should be written neatly and in order they were assigned.

A typical homework assignment is graded out of 20 points: 4 points for correctness of each problem. Bonus points result in additional credit.

0. (Ungraded)

- Finish reading Chapter 2 through section 2.4; this is what we covered 1st week. Did you find any mistakes or typos? If you did not, you might not have read carefully enough.
- Continue reading Chapter 2.

1. Problem 3 on page 46. [The order of the group is assumed to be at least 2.]

2. Let  $n \geq 3$  be an integer. Let  $S_n = A(\{x_1, x_2, \dots, x_n\})$ .

- (a) Show that  $S_n$  contains a subgroup  $H$  of index  $n$ . You must define  $H$  explicitly.
- (b) Find two elements  $\alpha, \beta \in S_n$  that are congruent modulo  $H$  (in the sense of definition on page 39), but that fail to satisfy  $\alpha^{-1}\beta \in H$ . [Hint: if stuck, try  $n = 3$  case first.]

3. Problem 21 on page 48.

4. Problem 27 on page 48.

5. Problem 19 on page 48.