# Algebraic Structures: homework #11
# Due 18 November 2024, at 9am via Gradescope

To receive full credit, all work must be shown. A passage means what careful but unimaginative reader thinks it does. Add details if in doubt. The problems should be written neatly and in order they were assigned.

A typical homework assignment is graded out of 20 points: 4 points for correctness of each problem. Bonus points result in additional credit.

0. (Ungraded)
   - Finish reading Section 5.1; this is what we covered by the 11th week. Did you find any mistakes or typos? If you did not, you might not have read carefully enough.
   - Continue reading Chapter 5.
1. Problem 4 on page 215.
2. Problem 5 on page 215.
3. Problem 9 on page 215. [ For (b), show that $F$ contains $J_p$. For (c), use Lagrange's theorem. ]
4. Let $p$ be a prime number, and $n$ is a positive integer. For a polynomial $f = a_0 + \cdots + a_n x^n \in J[x]$ denote $\overline{f} = b_0 + \cdots + b_n x^n \in J_p[x]$ its reduction modulo $p$, i.e., $b_i = a_i + pJ$.
   (a) Prove that if $f$ is monic (i.e., $a_n = 1$) and $\overline{f}$ is irreducible, then $f$ is irreducible.
   (b) Give a non-monic example (but still with $a_n \neq 0$) such that $\overline{f}$ is irreducible, but $f$ is reducible.
   (c) Give an example of a monic $f$ which is irreducible, but such that $\overline{f}$ is reducible.

For parts (b) and (c), you can choose prime $p$. You do not have to give examples for every $p$.

5. Test is on Wednesday. Free 4 points.